The Hybrid Local Ensemble Transform Kalman Filter

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Methodology

The Hybrid-LETKF scheme is formulated by combining the gain matrices rather than the covariances matrices as with traditional hybrid approaches

$$\hat{K} = \beta_1 K + \beta_2 K^B + \beta_3 K^B H K$$

$$K = P^b H^T \left(H P^b H^T + R \right)^{-1}$$

$$K^B = B H^T \left(H B H^T + R \right)^{-1}$$

$$x^a_{Hybrid} = \overline{x}^b + \hat{K}\left(y^o - H\overline{x}^b \right)$$

Choosing β_1=1, β_2=α, and β_3=-α, we can form an algebraically equivalent algorithm that combines ensemble means while utilizing existing software with virtually no modification. We call this particular algorithm the Hybrid/Mean-LETKF

$$\tilde{P}^a = \left[\frac{(k-1)}{\rho} I + \left(Y^b \right)^T R^{-1} Y^b \right]^{-1}$$

$$W^a = \left[(k-1)\tilde{P}^a \right]^{1/2}$$

$$X^a = X^b W^a$$

$$\overline{w}^a = \tilde{P}^a \left(Y^b \right)^T R^{-1} \left(y^o - \overline{y}^b \right)$$

$$\overline{x}^a = X^b \overline{w}^a + \overline{x}^b$$

LETKF

$$J(x^a) = \left(x^a - \overline{x}^a \right)^T \hat{B}^{-1} \left(x^a - \overline{x}^b \right) + \left(y^o - H x^a \right)^T \hat{R}^{-1} \left(y^o - H x^a \right)$$

3DVar

$$\overline{x}^a_{Hybrid} = \alpha x^a + (1-\alpha)\overline{x}^a$$

$$X^a_{Hybrid} = X^a + \overline{x}^a_{Hybrid} v^T$$

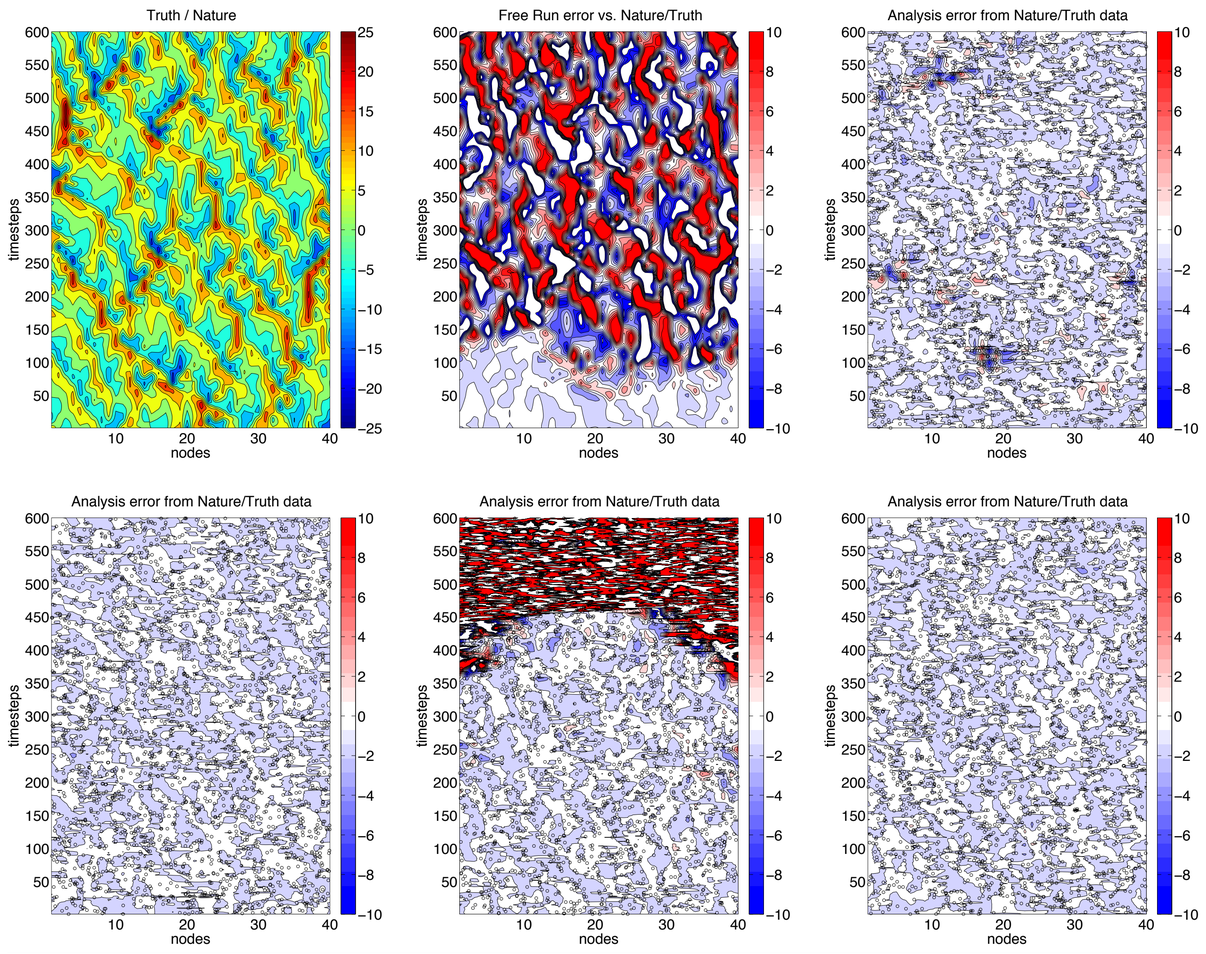
Form the hybrid analysis from a linear combination of the results from LETKF and the modified 3DVar. Then re-center the ensemble at this new mean state

The LETKF-Hybrid stabilizes filter divergence that is prevalent with small ensemble sizes and sparse observations

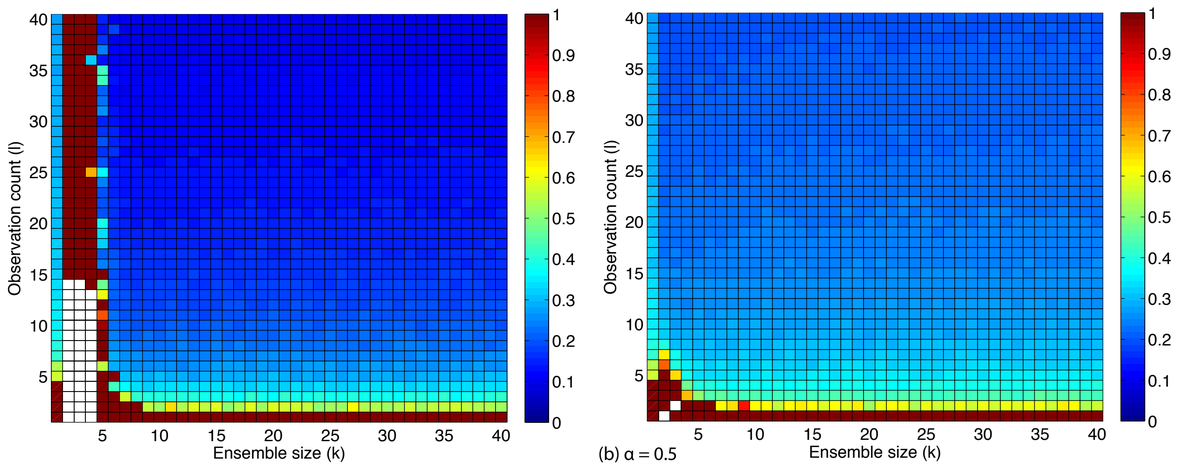
Results using the Lorenz-96 model

Results are shown first with the Lorenz-96 model (m=40,F=20), then with the GFDL MOM4p1 Ocean General Circulation Model

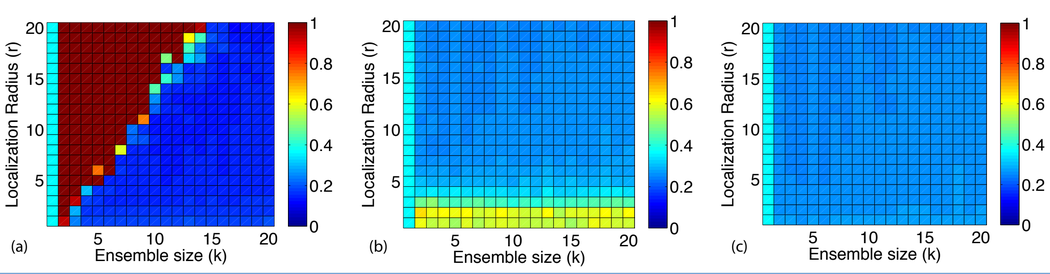
$$\frac{dx_j}{dt} = \left(x_{j+1} - x_{j-2} \right) x_{j-1} - x_j + F$$



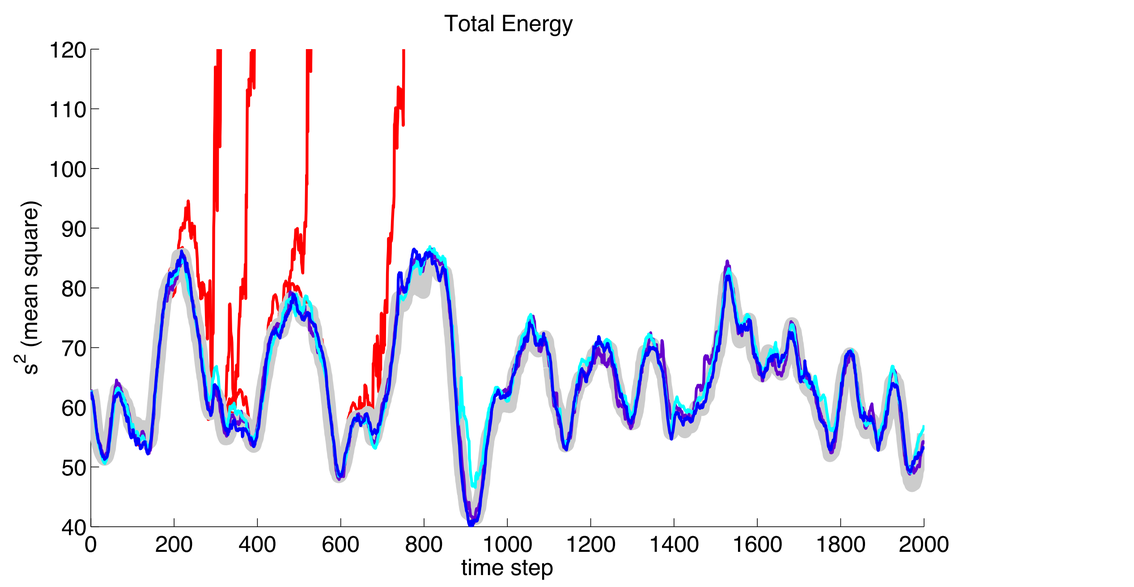
Nature run for Lorenz-96 over 600 time steps with Δt=0.01 (top left). Free Run error (top center). The following analyses are performed with l=4 observations per time step: Analysis error for 3D-Var (top right), LETKF, k=20 (bottom left), LETKF, k=5 (bottom center), and the Hybrid/Mean-LETKF, k=5 (bottom right).



Mean absolute analysis error for the (a) standard LETKF and (b) Hybrid-LETKF α=0.5, using ensemble sizes k=2..40, and observation coverage l=1..40 randomly chosen throughout the domain. Results at k=1 correspond to the standard 3D-Var and are identical in the subsequent figures. Empty squares indicate cases in which the Runge Kutta ODE solver could not reach the required tolerance.

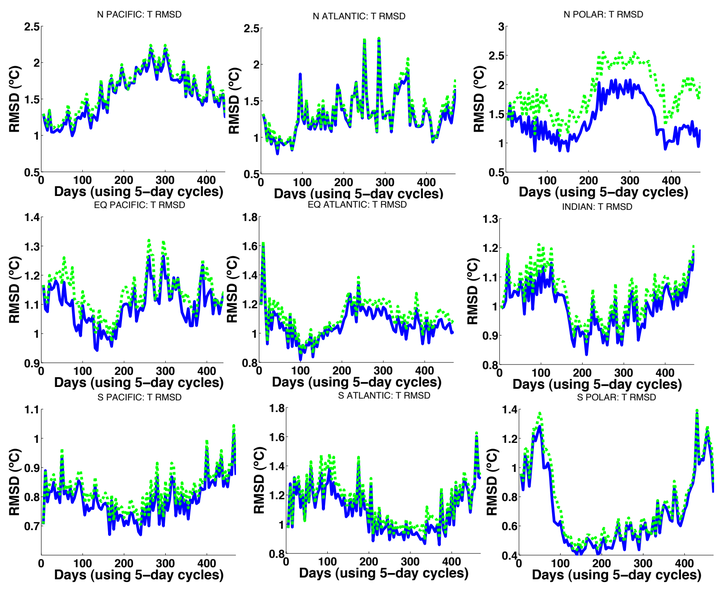


Mean absolute analysis error, with the localization radius varying from r=1 to r=20 while holding the observation count fixed at l=20 observations. Results are given for (a) LETKF, (b) the Hybrid/Mean-LETKF (α=0.5) with rB=r, and (c) the Hybrid/Mean-LETKF (α=0.5) using 3D-Var applied globally with rB=5.

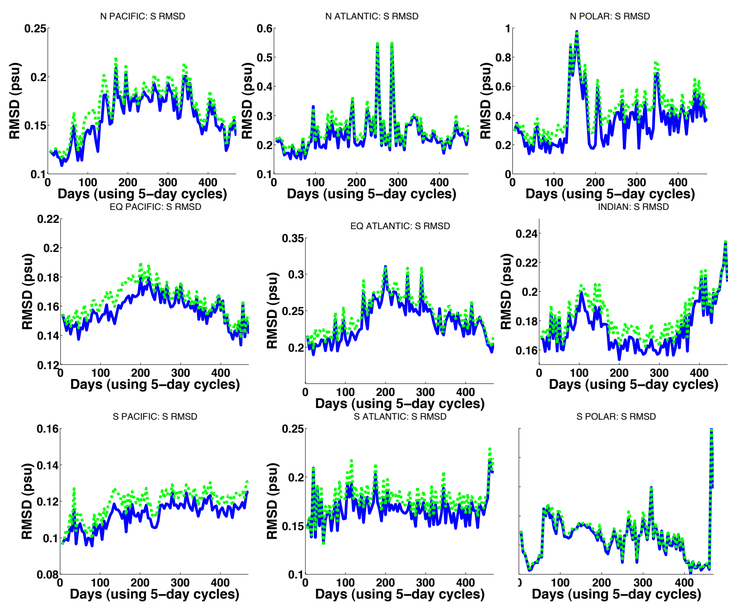


The total energy s^2 is plotted for 2000 time steps (100 days) for the ensemble mean state in an analyses using l=4 observations per time step. Standard LETKF is shown in gray (k=20) and cyan (k=6). Four different cases of standard LETKF (k=5) are shown in red, each blowing up at a different time due to randomness in observation locations. The Hybrid/Mean-LETKF (k=5), shown in blue, recovers the stability and accuracy of the standard LETKF (k > 5). The Hybrid/Covariance-LETKF is shown in violet (k=5).

Results using NCEP's Operational-Resolution GFDL MOM4p1 Ocean Model

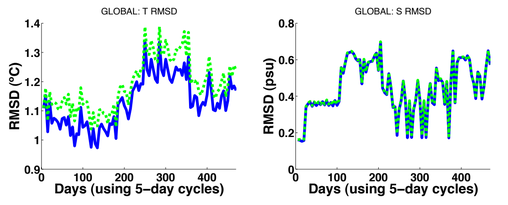


Temperature O-F RMSD



Salinity O-F RMSD

Two data assimilation schemes are compared, NCEP's operational 3DVar-based Global Ocean Data Assimilation System (3DVar-GODAS) and the Hybrid (Hybrid-GODAS). The Observed-minus-Forecast RMS deviations are shown for Temperature and Salinity for various regions throughout the global ocean.



Global O-F RMSD

The departure between the ocean temperature state after 5 months at 50m and in the top 500m at the equator