Correlations of Control Variables for Representing Forecast Errors on Cubed-Sphere Grids

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Introduction

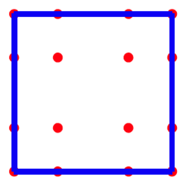
- The background error covariance is essential in data assimilation for spreading out information spatially especially in data-sparse areas, providing statistically consistent and dynamically balanced increments at the neighbouring grid points and levels of the model [1]. The full representation of the matrix is impossible because of the huge size typically 10^7 size more, so the matrix is constructed implicitly by means of a variable transformation to make B matrix be diagonal in control variable space.
- Background error covariance can be modeled by control variable transforms with balance operators which specify dynamic constraints in an atmospheric balance relationship. Balance operators were developed based on the equations of fluid motions (η-coordinate primitive equation).
- The statistical structure of cross-correlations of control variables ($\psi, \chi, M_u, q, P_{su}$) will be presented.

Methodology

Forecast Error Sample

- Forecast error statistics were based on Community Atmosphere Model-Spectral Element (CAM-SE) model runs every 6 hours with 64 ensemble members.
- CAM-SE is built upon the cubed-sphere grid, where the grid points are located at Legendre-Gauss-Lobatto (LGL) points on each local element of 6 faces on the sphere.
- We used the cubed-sphere geometry based on the spectral element method which is better for parallel application to apply control variable transform.
- CAM-SE ne16 np4 L30 / 64 ensemble members

Figure was created by Peter Lauritzen(NCAR) and Dennis et al.(2012) from http://earthsystemcog.org/projects/dcmip-2012/cam-se



Control Variable Transformation

- The variable transformation from model variables to a set of control variables whose errors were assumed to be uncorrelated was developed on the cubed sphere-using Galerkin method [2]. The motivation of the control variable transform is to capture the properties of B without the need for an explicit matrix [3].
- Winds were decomposed into rotational part and divergent part by introducing stream function and velocity potential as control variables(Helmholtz' decomposition). The dynamical constraints for balance between mass and wind were made by applying linear/nonlinear balance operators.
- The balanced fields can be derived from the tangent linear equations in the hybrid vertical coordinate.
- With some approximation, a mass variable δM has geopotential term and pressure gradient term.

Primitive equation η-coordinate

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\zeta + f)\boldsymbol{k} \times \boldsymbol{V} + \nabla\left(\frac{1}{2}u^2 + \Phi\right) + \dot{\eta}\frac{\partial \boldsymbol{V}}{\partial \eta} + \frac{RT_v}{p}\nabla p = 0$$

$$\delta M = \delta\Phi + \frac{RT_r}{\bar{p}}\delta p$$

Forecast Error Covariance

$\delta x = [\,\delta\psi \;\; \delta\chi \;\; (\delta\phi, \delta p_s) \;\; \delta q\,]^T$

$\delta x = \mathbf{K}\delta x_u$

$\mathbf{B} = \mathbf{K}\mathbf{B}_u\mathbf{K}^T$

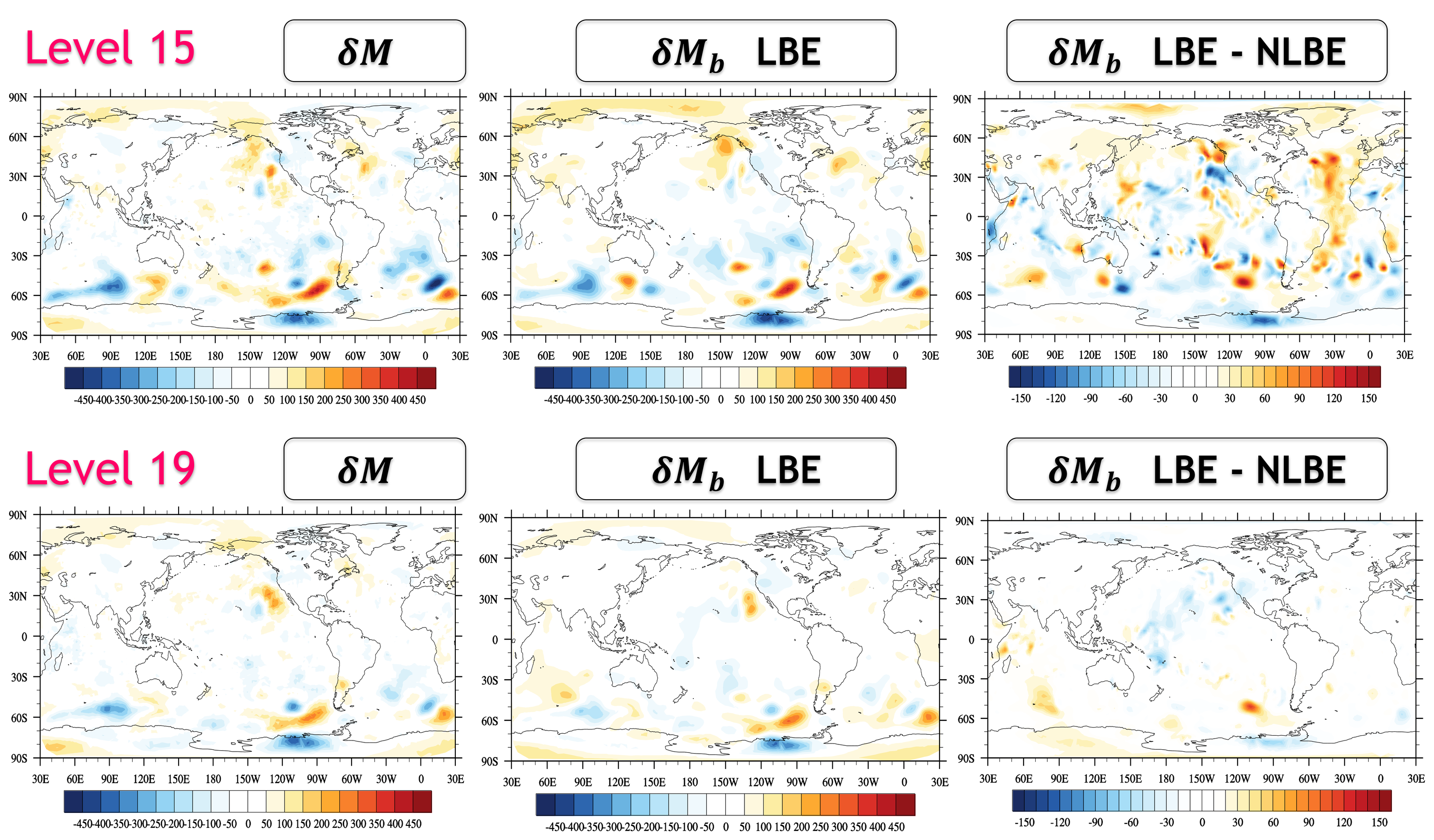
$\mathbf{B}_u$: block-diagonal, cross-covariance in $\mathbf{B}_u$ are 0

Balance Operator

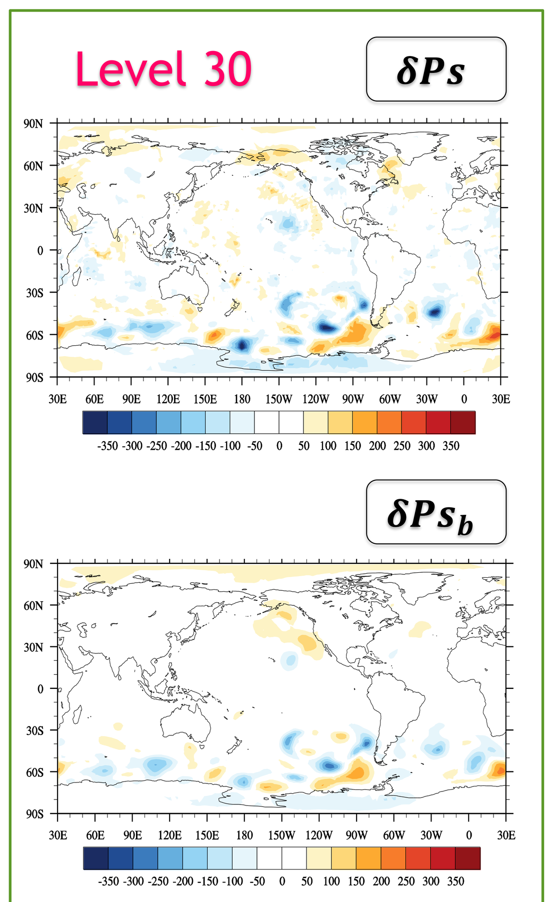
Correlation

Helmholtz decomposition	$\delta v = \nabla\delta\chi + k \times \nabla\delta\psi$
Linear balance (LBE)	$-fk \times \delta V = \nabla\left(\delta\Phi + \frac{RT_r}{\bar{p}}\delta p\right)$
Nonlinear balance (NLBE)	$-(fk \times \delta V + (V\cdot\nabla\delta V + \delta V\cdot\nabla V) = \nabla\left(\delta\Phi + \frac{RT_r}{\bar{p}}\delta p\right)$
Hydrostatic balance	$\Phi_s + \int_\eta^1 \frac{R_{gas}T_v}{p}\frac{\partial p}{\partial \eta} = \Phi$
Balanced Ps	$\delta P_{s,b} = \frac{p_s}{RT_{ref}}\nabla_\eta^{-2}[\nabla_\eta \cdot (f\nabla_\eta\delta\psi)]$

Results - Balanced fields



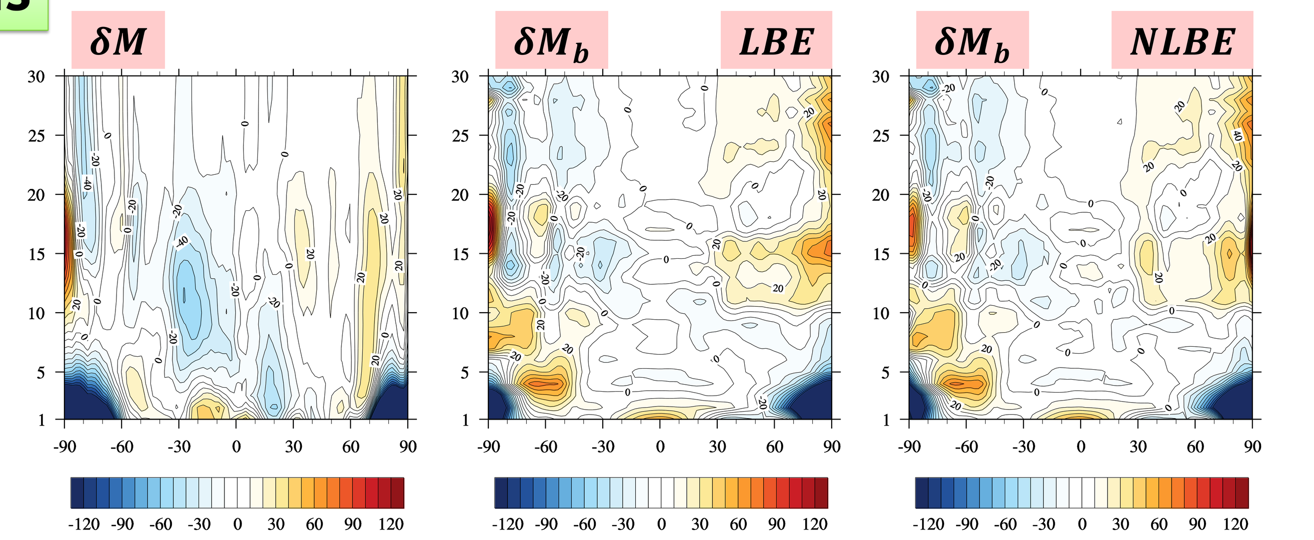
- The synoptic, horizontal structure of model mass variable is reproduced quite well by the balanced field.
- Some disagreement in horizontal structures of balanced model mass variable is shown quite large at model level 15.



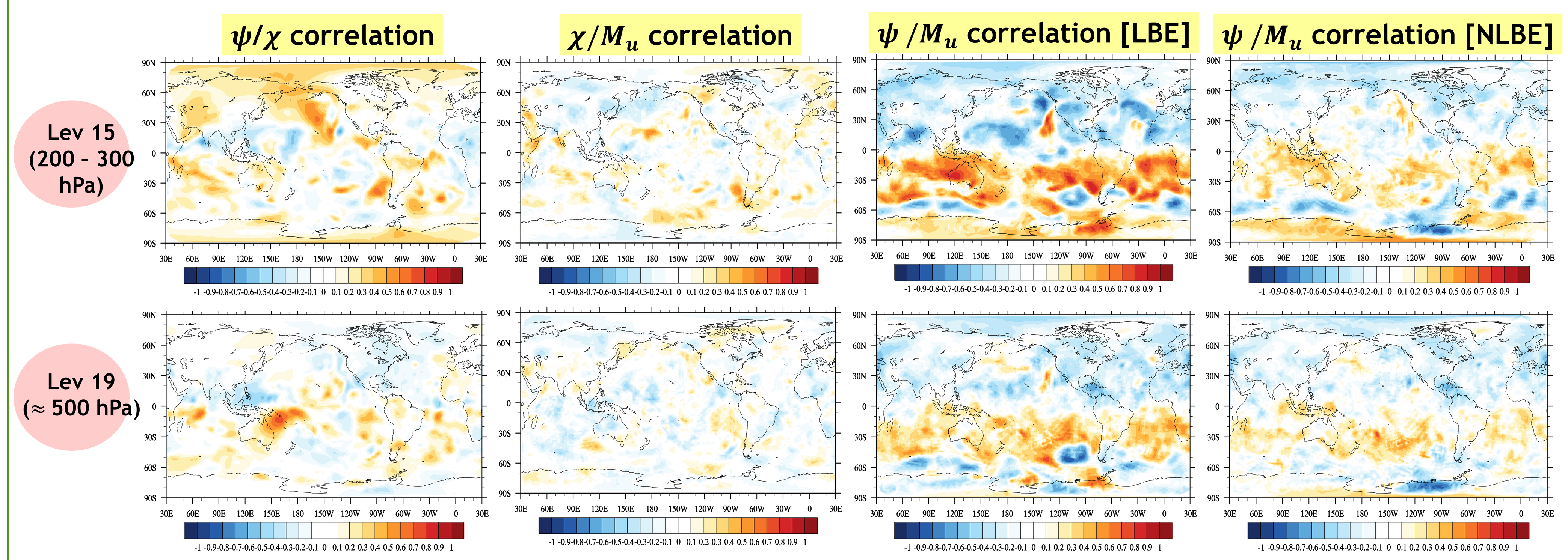
- The balanced Ps field contains much of the features as the model Ps field.

Results - Balanced fields

- Mass variable δM and δM_b from LBE and NLBE.
- Summation was over longitude. X is latitude y is model level (1:top, 30:bottom).
- Balanced parts are dominant in extra tropics in both LBE and NLBE cases.
- Vertical balanced mass structures are shown similar, but some disagreement between LBE and NLBE cases at model level 10~17 ($\approx$ 100~400 hPa).

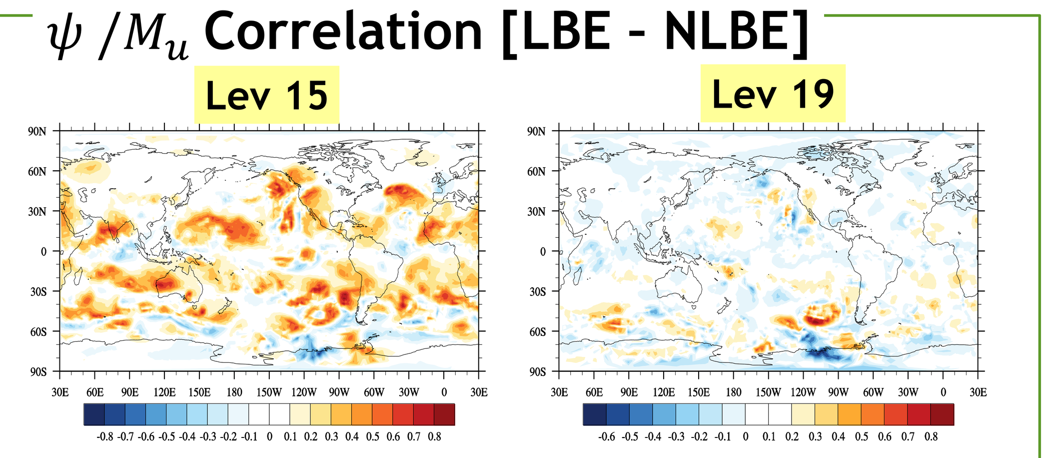


Results - Correlation

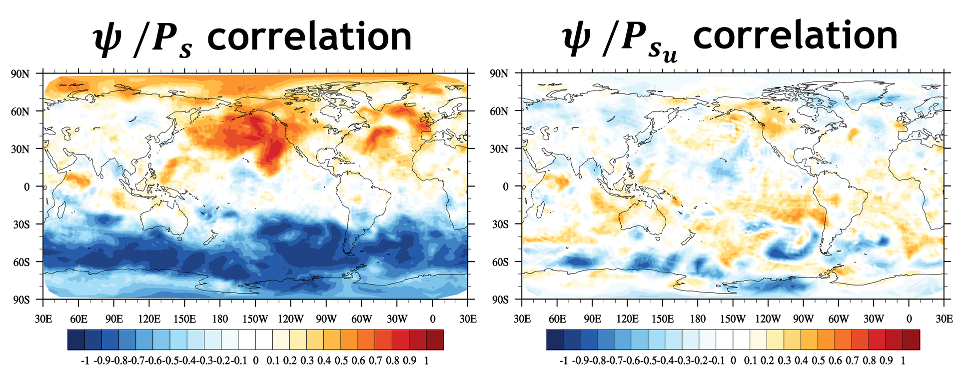


- Compared to correlations between model variables[not shown], correlations of control variables are shown to be reduced.
- ψ/χ correlation needs to be removed. Other operational NWP centers(Met Office, ECMWF(η_u), NCEP etc.) used unbalanced velocity potential by regression or dynamic constraints.
- By using NLBE operator, correlations between mass and winds comparably reduced than LBE case.
- Geostrophic approximation is dominant at model level 19 (about 500 hPa).
- NLBE with advection terms contribute to balance parts relating flow dependent terms such as strong curvature and jet etc. at model level 15.

ψ/M_u Correlation [LBE - NLBE]

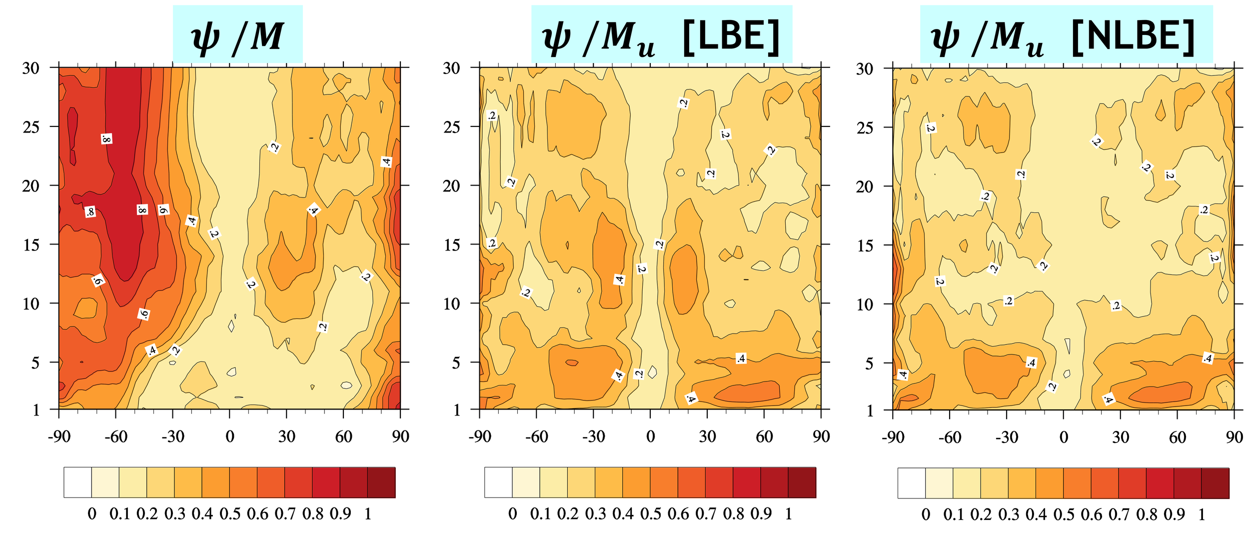


- ψ/M_u correlations were compared using absolute correlation using LBE and NLBE.
- By using NLBE, mass/winds correlation is reduced as much as 0.8 at model level 15.



- Balanced surface pressure was calculated using winds at model level 30.
- Correlation between winds and unbalanced surface pressure(P_{s_u}) is to be smaller than correlation between winds and P_s.

ψ/M_u Correlation [vertical structure]



- Vertical structures of ψ/M_u correlation were compared using absolute correlation using LBE and NLBE. X is latitude y is model level (1:top, 30:bottom).
- ψ/M_u correlation using NLBE was much more reduced than that of LBE over all levels except model level 1-5.

Conclusion

- Control Variable Transform by balance operator was to remove cross-correlation between variables.
- Balanced variables well captured most of the synoptic and horizontal features of model.
- Mass/wind correlation was well reduced by only using linear balance operator at model level 19, however, NLBE contributed to much smaller correlation by well capturing balances associated with flow dependent terms above model level 15(except level1-5).
- ψ/χ Correlation needed to be removed by distinguishing balanced/unbalanced part.
- The development of balance operator associated with a moisture variable using statistical or physical methods is a future work.

References

[1] J. Derber and F. Bouttier. "A reformulation of the background error covariance in the ECMWF global data assimilation system," Tellus, vol. 51A, no. 2, pp. 195-221, 1999.

[2] Song, H. J and J. Kwun. "Development of control variable transformation on the Cubed Sphere Using the Continuous Galerkin Method," 2012 Solution of Partial Differential Equations on the Sphere, http://www.newton.ac.uk/programmes/AMM/Song.pdf, 2012.

[3] R. N. Bannster. "A review of forecast error covariance statistics in atmospheric variational data assimilation. II: Modelling the forecast error covariance statistics," *Quarterly Journal of the Royal Meteorological Society*, vol. 134, no. 637, pp. 1971-1996, October 2008.