Adaptive Localization in Ensemble Kalman Filter Methods by Controlling the Observation Space

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Introduction

Why adaptive localization?

In geophysical systems, the state dimension is very large. For these systems, Ensemble Kalman Filter Methods need to be localized to get result.

Tuning the localization parameter is very expensive. The dependence on other parameters is still unknown and the localization need to be retuned every time the model parameter are changed.

Are there some underlying relations between the assimilation parameters that can be used when setting up the system?

To answer this question the dependence of the localization radius and the ensemble size is examined in the case of very dense observations (e.g. sea surface height)

The aim is to reduce the effort to calibrate a data assimilation system and improve the assimilation performance.

Outline

The relation between the ensemble size and the localization radius was examined for different parameter settings and different models with increasing complexity.

Twin experiments for various different localization radii and ensemble sizes are computed (see. Tab.1) and averaged over several runs.

Three different weighting functions were used:

- Uniform weighting (DL)
- The 5th order polynomial (OL)
- Exponential weighting (EXP)

The results were classified into "converged" and "diverged" results, where a result was defined as diverged if the mean RMS error was larger than the observational error.

Since the results of L96 and SWE are very similar, only the latter is shown here.

	L96	SWE	FESOM
Nonlinear	Yes	Yes	Yes
Variables	1	3	10
Dimension	96	≈ 20.000	$\approx 10^7$
Filter	LETKF	LETKF	LETKF
Ensemble size	$5-40$	$8-40$	32
Weight function	DL/OL/EXP	DL/OL	OL
Inflation	1.05	1.05	1.1
Observation dim.	96	6400	68000
Localization radius	$0-40$	$20km-340km$	variable

Tab.1: Model and Filterparameter for the Lorenz-96 (L96) a shallow water model (SWE) and the global ocean model (FESOM).

Results - Shallow Water Model

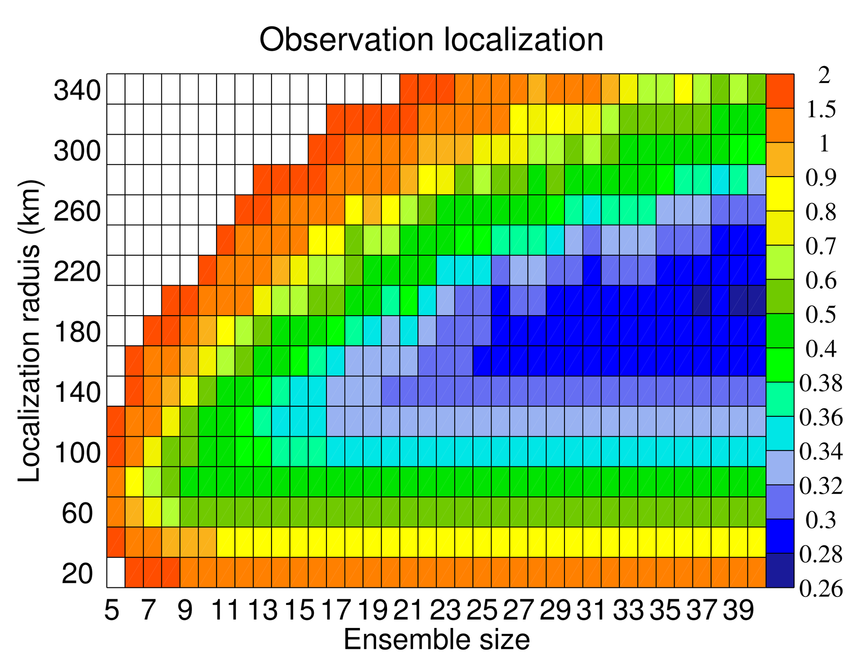
The mean RMS error for the different configurations shows a regular pattern. This did not change for both the L96 and the SWE model.

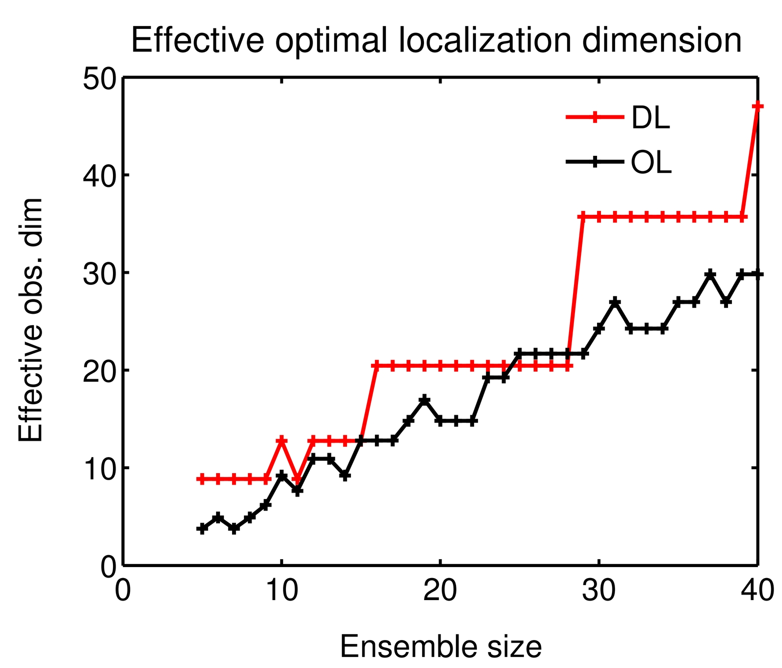
When a uniform weight is used to weight the observations for DL, the relation between the optimal localization radius and the localization radius is linear. This does not hold if the exponential or the 5th order polynomial are used.

But the results can be linked if instead of the localization radius an **effective observation dimension** is considered. This number is defined as the sum of all observation weights. It turns out, that this number behaves similar regardless of the weight function used.

An explanation is that the ensemble can only fit as many observations as there are degrees of freedom inside the ensemble. This does not change if the weights of the observations is reduced, but the degrees are distributed over more observations.

This motivates an adaptive localization radius that can be defined, by choosing the localization radius, so that the effective localization radius is equivalent to the ensemble size.





Results - Global ocean model

An adaptive localization radius was implemented in the FESOM model using the PDAF ([2], http://pdaf.awi.de)

The spatially varying localization radius was compared against two fixed localization radii (1000km and 500km).

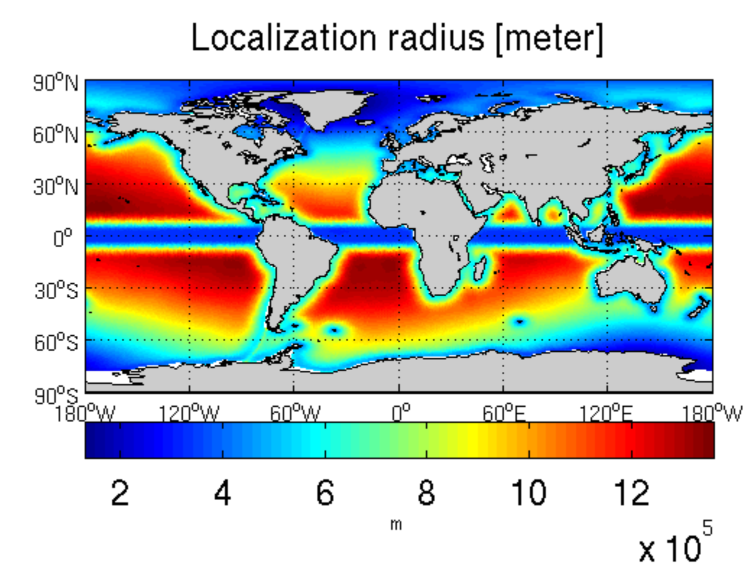
Synthetic observations of the full sea surface field have been assimilated,

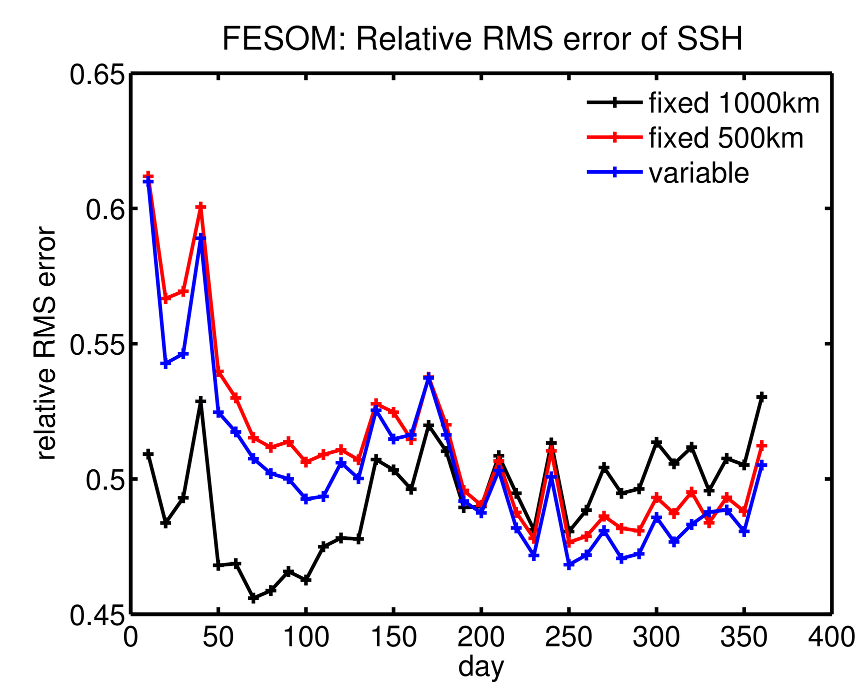
The large localization radius (1000km) reduces the RMS very quickly, but is increasing again towards the end of the assimilation experiment.

The results with the smaller and the adaptive localization radius are similar and, in contrast to the larger localization radius, the relative RMS is decreasing over time.

The relative RMS with the varying localization radius is slightly reduced the RMS compared to the fixed one.

An advantage of the adaptive localization is that there is no need to tune the localization





Sampling Quality

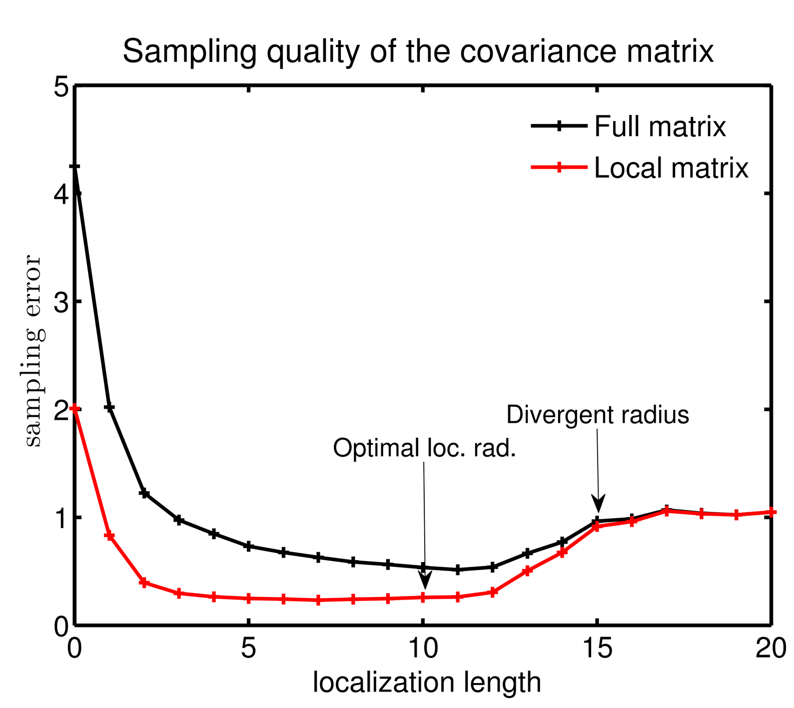
We investigated whether the optimal localization radius is consistent with the best approximation of the covariance matrix for the L96 model.

This was done by calculating the sampling error of the analysis covariance matrix. We considered two cases, the whole covariance matrix, and the part that is considered in the analysis.

If the localization radius is too small, the approximation of the covariance matrix is bad.

The best approximation of the covariance matrix coincides with optimal localization radius.

If the analysis is diverged, the local analysis gets as bad as a global one.



The error of the full and the partial covariance matrix that is used in the analysis.

Conclusion

From the experiments using the toy models it became clear, that the optimal localization radius in the Ensemble Kalman Filter is dependent on the ensemble size and the observation density.

For the toy models investigated in this study, there is an approximately linear relationship between the ensemble size and the optimal localization radius.

If a different weighting function is used, the optimal localization radius was achieved when the effective localization radius is in the order of the ensemble size.

An Ensemble Kalman Filter using this estimate to define an adaptive localization radius performed comparable to an optimal tuned localization radius on a global ocean model. These result was achieved without further tuning.

References

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