Data Assimilation Experiments using the Back and Forth Nudging and NEMO OGCM

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Summary

The Diffusive Back and Forth Nudging (DBFN) is an easy-to-implement iterative data assimilation method based on the well-known Nudging method. It consists in a sequence of forward and backward model integrations, within a given time window, both of them using a feedback term to the observations. Therefore in the DBFN, the Nudging asymptotic behavior is translated into an infinite number of iterations within a bounded time domain. In this method, the backward integration is carried out thanks to what is called backward model, which is basically the forward model with reversed time step sign. To maintain numeral stability, the diffusion terms also have their sign reversed, giving a diffusive character to the algorithm. In this presentation, the DBFN performance to control a primitive equation ocean model is investigated. In this kind of model, non-resolved scales are modelled by diffusion operators which dissipate energy that cascades from large to small scales. Thus, in this article the DBFN approximations and their consequences on the data assimilation system set-up are analysed. Our main result is that despite the lack of accuracy of the backward model, the DBFN provides results that are comparable to those produced by a 4Dvar implementation. The required conditions include the use of a short assimilation window ($\simeq$ 10 days) along with a reduced model diffusion and a Nudging gain able to spread the observation information to the non-observed variables.

Diffusive Back and Forth Nudging algorithm

The BFN was first introduced by Auroux and Blum (2008) and consists in an iterative algorithm which sequentially solves the forward model equations with a feedback term to the observations and the backward model equations with the sign of the feedback term reversed. The initial condition of the backward integration is the final state obtained after integration of the forward nudging equation. At the end of each iteration one obtains an estimate of the initial state of the system. The iterations are carried out until convergence.
We used the Diffusive Back and Forth Nudging-DBFN (Auroux et al., 2011), designed to treat the instabilities of the backward integration in dissipative systems. In the DBFN we keep the sign of the diffusion, during the backward integration, consistent with the forward model and only the non-diffusive physical model is solved backwards. This is of relevant interest in oceanography because the non-diffusive part of the model is generally reversible. We assume that the time continuous model satisfies dynamical equations of the form:

$$\frac{dX}{dt} = F(X) + \nu \Delta X, \qquad 0 < t < T, \tag{1}$$

with initial condition $X(0) = x_0$, where F denotes the nonlinear model operator without diffusive terms, ν is the diffusion coefficient and Δ represents the diffusion operator. In the following we will denote by H the observation operator, allowing one to compare the observations $X_{obs}(t)$ with the corresponding $H(X(t))$, the subspcript k is the iteration index and K and K' are the forward and backward gain matrices respectively. If we apply nudging to forward system (1) we obtain:

$$\frac{\partial X_k}{\partial t} = F(X_k) + \nu \Delta X_k + K(X_{obs} - H(X_k)) \tag{2a}$$
$$X_k(0) = \tilde{X}_{k-1}(0), \qquad 0 < t < T, \tag{2b}$$

while nudging applied to the backward system gives:

$$\frac{\partial \tilde{X}_k}{\partial t} = F(\tilde{X}_k) - \nu \Delta \tilde{X}_k - K'(X_{obs} - H(\tilde{X}_k)) \tag{3a}$$
$$\tilde{X}_k(T) = X_k(T), \qquad T > t > 0. \tag{3b}$$

Using the variable transformation $t' = T - t$, we can write the backward model as:

$$\frac{\partial \tilde{X}_k}{\partial t'} = -F(\tilde{X}_k) + \nu \Delta \tilde{X}_k + K'(X_{obs} - H(\tilde{X}_k)) \qquad \tilde{X}_k(t'=0) = X_k(T).$$

This equation shows that the backward equation can be solved with an initial condition and the same diffusion term as in the forward equation.
The convergence criterium we use in the following is given by the inequality:

$$\frac{\|X_k(t=0) - X_{k-1}(t=0)\|}{\|X_{k-1}(t=0)\|} \leq \varepsilon$$

where $\varepsilon = 0.005$ (based on sensitivity tests).

Partial Least Squares regression (PLS)

The PLS method was first introduced by Wold (1975) to address the problem of econometric path modeling, and was subsequently adopted for regression problems in chemometric and spectrometric modeling. In the method description, $X \in \mathbb{R}^{n \times M}$ is considered as the observed or predictor variables and $Y \in \mathbb{R}^{n \times N}$ as the non-observed or response variables. In our notation n is the sample size and M and N are respectively the size of the state space of X and Y. Besides, X and Y are centered and have the same units. The PLS regression features two steps: a dimension reduction step in which the predictors from matrix X are summarized in a small number of linear combinations called "PLS components". Then, these components are used as predictors in the ordinary least-squares regression. The PLS as well as the principal component regression can be seen as methods to construct a matrix T of p mutually orthogonal components defined as linear combinations of X:

$$T = XW,$$

where $T \in \mathbb{R}^{n \times p}$ is the matrix of new components, and $W \in \mathbb{R}^{M \times p}$ is a weight matrix satisfying a particular optimality criterium.
The columns $w_1; \ldots; w_p$ of W are calculated according to the following optimization problem:

$$w_i = \arg\max_{w} \{cov(Xw, Y)^2\}$$

subject to $w_i^T w_i = 1$ and $w_i^T X^T X w_j = 0$ for $j = 1, \ldots, i-1$.
The PLS estimator $\hat{B}^{PLS}$ is given by:

$$\hat{B}^{PLS} = W(W^T X^T X W)^{-1} W^T X^T Y.$$

An immediate consequence is that when $W = I$ the original least square solution is obtained. The number of components p is chosen from cross-validation. This method requires a test of the model with objects that were not used to build the model. The data set is divided into two contiguous blocks; one of them is used for training and the other to validate the model. Then the number of components giving the best results in terms of mean residual error and estimator variance is sought.

Results

The errors in the initial condition decrease exponentialy during the iterative procedure for both observed and non observed variables. The way they decrease depends on the gain factor **K** and the information content available from the observations. The smaller the number of observations the bigger the number of iterations required to converge, although this does not mean that the final states are the same, even if the observations are extracted from a model solution.

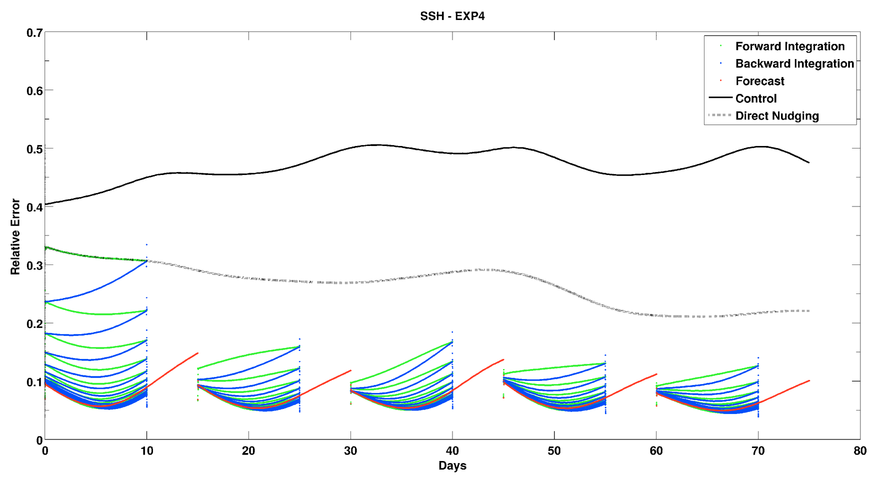

Figure 1: In color: evolution of the errors during the Back and Forth iterations and during the forecast phase. In black: evolution of the error for the control and direct nudging experiments.

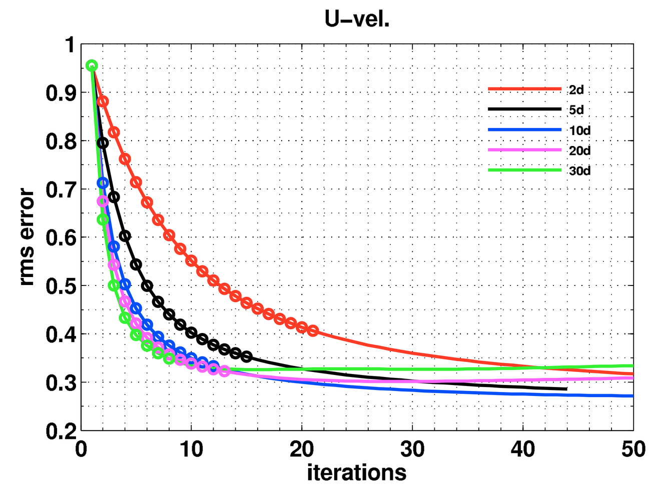

Figure 2: Relative errors on the initial condition with respect to the iterations for the experiment assimilating daily gridded SSH fields.

The DBFN converges to good initial condition estimates:

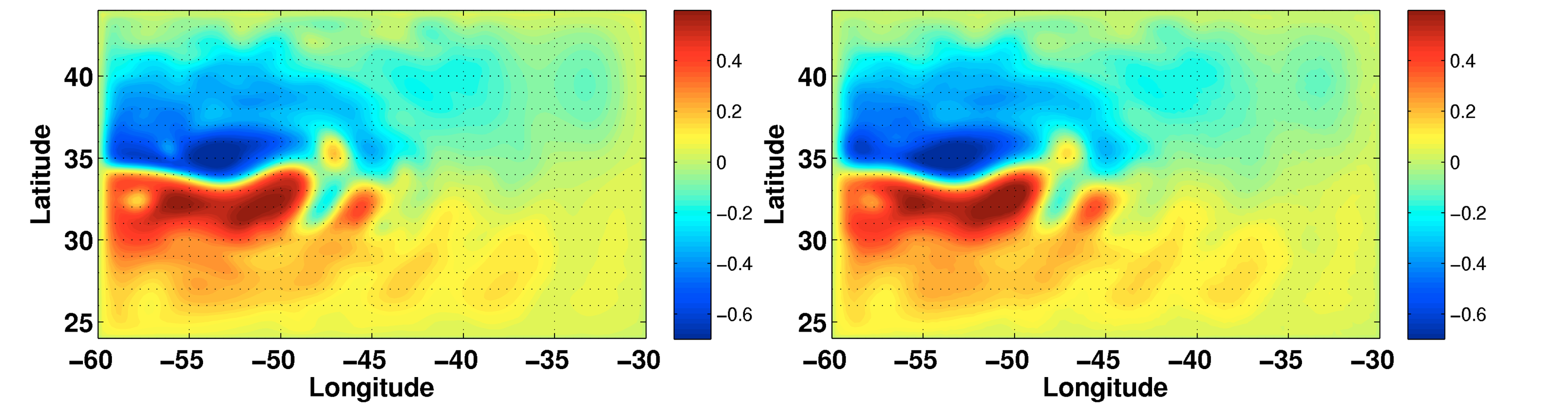

Figure 3: Example of model run: comparison between the true state (left) and identified state by the DBFN (right).

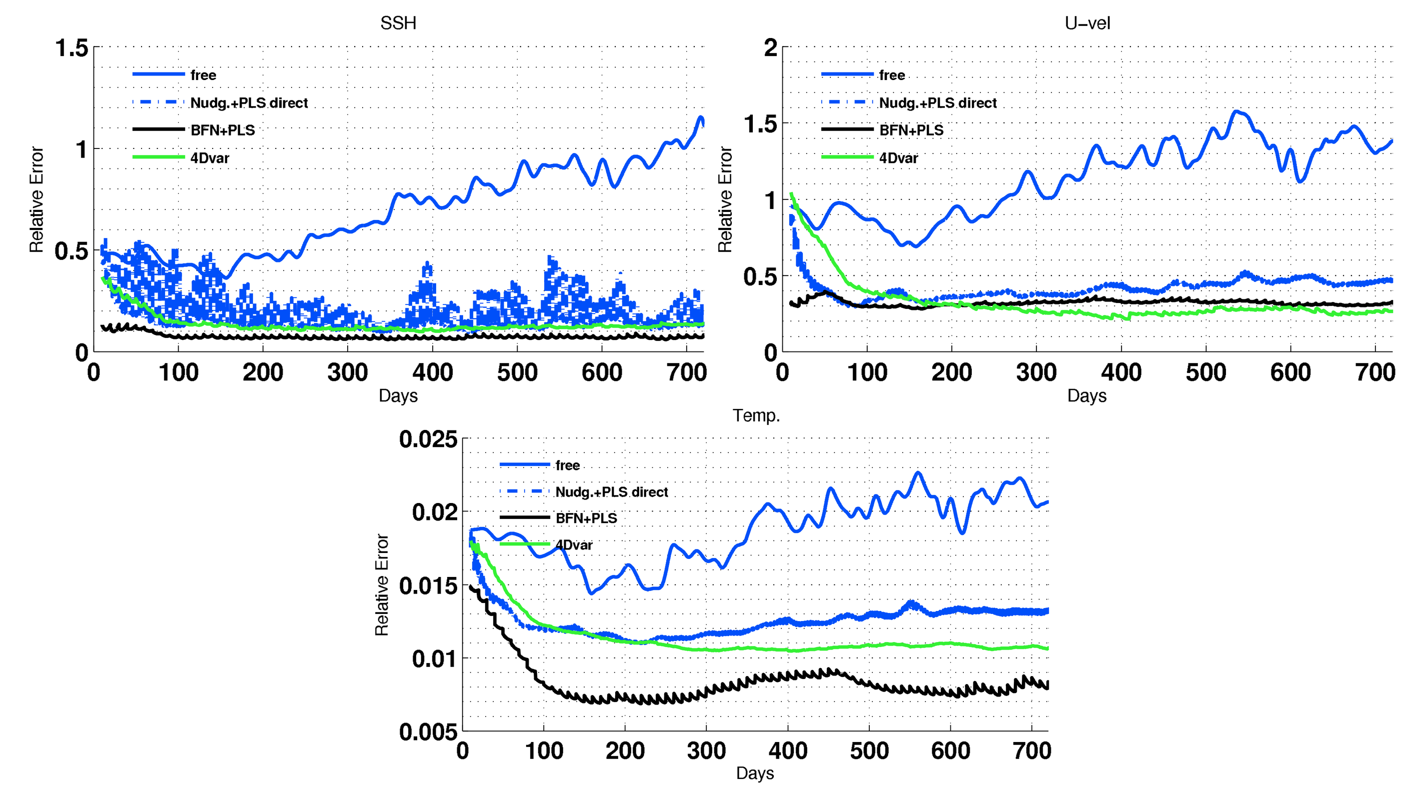

Figure 4: Relative error of the SSH, U-velocity and Temperature

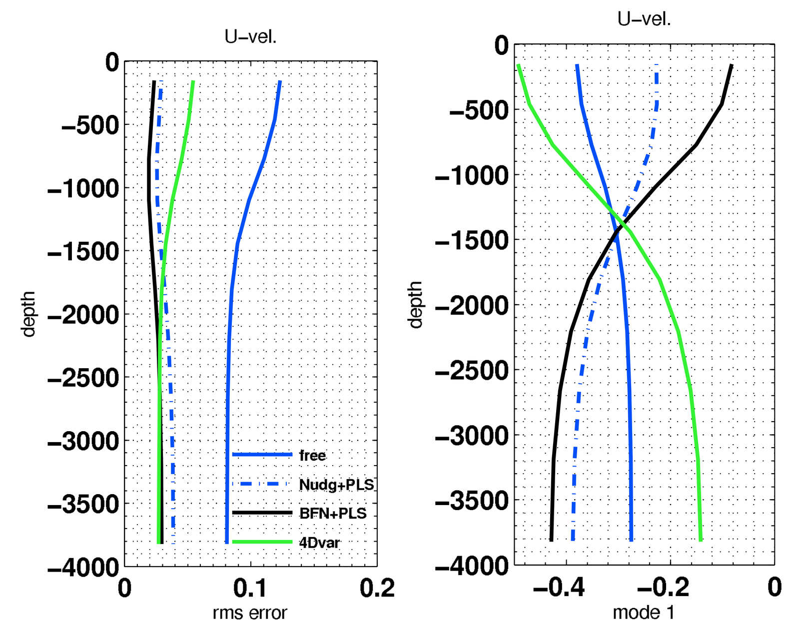

Figure 5: RMS of vertical zonal velocity and EOF error modes calculated using forecast from day 200 to day 720.

Conclusions

- DBFN can be used for ocean DA despite the low accuracy of the backward integration.
- Use of scalar gains requires high spatial and temporal availability of data.
- In the case of sparse data, the PLS model builds complex functions that propagate the information from the data to the non-observed variables and non-observed regions of the domain.
- DBFN results are at least comparable with 4DVar, with a much lower computational cost.

References

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