

B-Preconditioned Minimization Algorithms for Variational Data Assimilation with the Primal and Dual Formulations

Selime Gürol^{a, b}, Anthony T. Weaver^c, Serge Gratton^{a, d}, Andrew M. Moore^e,
Andrea Piacentini^c and Hernan G. Arango^f

^a CERFACS, France, ^b ECMWF, UK, ^c CERFACS / SUC URA 1875, France, weaver@cerfacs.fr,
^d ENSEEIHT, France, ^e Department of Ocean Sciences, University of California at Santa Cruz, USA,
^f Institute of Marine and Coastal Sciences, The Rutgers University of New Jersey, USA

Variational data assimilation (Var) problems in meteorology and oceanography require the solution of a regularized nonlinear least-squares problem. Practical solution algorithms are based on the incremental approach which involves the iterative solution of a sequence of linear least-squares (quadratic minimization) sub-problems. Each sub-problem can be solved using a primal approach where the minimization is performed in a space spanned by vectors of the size of the model control vector, or a dual approach where the minimization is performed in a space spanned by vectors of the size of the observation vector. The dual formulation can be advantageous for two reasons. First, the dimension of the minimization problem with the dual formulation does not increase when additional control variables, such as those accounting for model error in a weak-constraint formulation, are considered. Second, whenever the dimension of observation space is significantly smaller than that of the model control space, the dual formulation can reduce both memory usage and computational cost.

This presentation describes recent work in developing the dual approach for two operational ocean Var systems. One is a global ocean 3D-Var system for the NEMO model, while the other is an eddy-resolving regional 4D-Var system for the ROMS model. NEMO employs the Restricted **B**-preconditioned Conjugate Gradient (RBCG) method ([2]), while ROMS employs the Restricted **B**-preconditioned Lanczos (RBLanczos) method ([4]). RBCG and RBLanczos, and the corresponding **B**-preconditioned Conjugate Gradient (BCG) and Lanczos (BLanczos) algorithms used in the primal approach, generate mathematically equivalent iterates and require the same number of applications of the standard operators \mathbf{H} , \mathbf{H}^T , \mathbf{B} and \mathbf{R}^{-1} needed in Var. Furthermore, all these algorithms can be implemented without the need for a square-root factorization of \mathbf{B} , which is convenient with general \mathbf{B} formulations such as those used with ensemble and hybrid ensemble Var. Numerical results comparing the dual and primal algorithms in NEMO and ROMS will be presented. Extensions of this work to account for multiple outer-loop iterations and second-level preconditioners based on Limited-Memory Preconditioners such as Quasi-Newton and Ritz ([1], [3]) will also be described.

References

- [1] Gratton, S., Gürol, S., and P. L. Toint, “Preconditioning and globalizing conjugate gradients in dual space for quadratically penalized nonlinear least squares problems”, *Comput. Optim. Appl.*, vol. 54, pp. 1-25, 2013.
- [2] Gratton, S. and J. Tshimanga, “An observation-space formulation of variational assimilation using a Restricted Preconditioned Conjugate-Gradient algorithm”, *Q. J. R. Meteorol. Soc.*, vol. 135, pp. 1573-1585.
- [3] Gürol, S. “Solving regularized non-linear least squares problems in dual space with application to variational data assimilation”, PhD thesis, Institute National Polytechnique de Toulouse, June 2013.
- [4] Gürol, S., Weaver, A. T., Moore, A. M., Piacentini, A., and H. G. Arango. “B-preconditioned minimization algorithms for variational data assimilation with the dual formulation”, *Q. J. R. Meteorol. Soc.*, in press, 2013.