Performance Bounds for Particle Filters Using the Optimal Proposal

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Particle filters may be cast in the form of sequential importance sampling and thus allow the choice of a proposal density. This choice is known to be crucial to the performance of the algorithm. We show that the asymptotic arguments of Bengtsson et al. (2008) and Snyder et al. (2008) apply directly to the "optimal" proposal density, in which new particles are drawn at each observation time from a distribution that is conditioned on the new observations and which minimizes the variance of the particle weights over different realizations of particles drawn from the proposal density. The asymptotic results demonstrate that the ensemble size required for the optimal proposal to avoid degeneracy of the particle-filter update (i.e., the situation in which a single particle receives a weight very close to 1) will grow exponentially with an appropriately defined measure of the problem size. At the same time, the asymptotic results indicate that the ensemble size needed may be dramatically smaller for the optimal proposal than for the standard proposal, in which new particles are drawn by evolving particles from the previous time under the system dynamics. For linear, Gaussian systems, an explicit expression for the exponent can be derived and the exponent for the standard proposal is always larger than that for the optimal proposal. The degree to which a smaller ensemble is feasible with the optimal proposal depends crucially on the magnitude of the system noise.

References:

Bengtsson, T., P. Bickel, and B. Li, 2008: Curse of dimensionality revisited: Collapse of the particle filter in very large scale systems. ***Probability and Statistics: Essays in Honor of David A. Freedman,*** **D. Nolan and T. Speed, Eds., Vol. 2, Institute of Mathematical Statistics, 316–334.**

Snyder, C., T. Bengtsson, P. Bickel and J. Anderson, 2008: Obstacles to high-dimensional particle filtering. ***Mon. Wea. Rev.,*** **136, 4296-4640.**