Lagrangian Data Assimilation: A Hybrid Particle-Ensemble Kalman Filter and Applications to Geophysical Fluid Flows

Laura Slivinski[a], Bjorn Sandstede[a], Elaine Spiller[b] and Amit Apte[c]

[a] *Division of Applied Mathematics, Brown University, USA, laura_slivinski@brown.edu,* [b] *Department of Mathematics, Statistics and Computer Science, Marquette University, USA,* [c] *TIFR Centre for Applicable Mathematics, Bangalore, India.*

Lagrangian data assimilation involves using observations of the positions of passive drifters in a flow in order to obtain a probability distribution on the underlying Eulerian flow field; for example, positions of rafts released into the ocean can be assimilated in order to estimate the currents in the ocean. This framework has several complications that differ from some traditional data assimilation frameworks. First, as in many geophysical applications, the dimension of the flow variable is very high (10^6). However, unlike many atmospheric applications, the dimension of the observations (e.g. the drifter variable) is relatively low dimensional; if we have, say, 10 drifters, then the dimension of the drifter variable would be 20. The second main complication is the highly nonlinear, chaotic nature of the Lagrangian trajectories of the drifters. Even if the flow itself evolves linearly, the evolution of the drifters will be highly nonlinear.

These complications lead to disadvantages with traditional sequential data assimilation methods. The first traditional method is the ensemble Kalman filter (EnKF). This method works well for high-dimensional problems when localization is included, and when the true posterior distribution is close to Gaussian. However, its disadvantage becomes clear when dealing with strongly non-Gaussian distributions (which arise from the nonlinear evolution of the drifters): the posterior distribution of the EnKF will be closer to a Gaussian than the true (Bayes) posterior. The second traditional method considered here is the particle filter. This method approximates the true Bayes posterior when enough particles are used; however, the necessary number of particles has been shown to increase exponentially in a variable related to the system dimension [1]. Therefore, the particle filter is intractable when applied to a problem with a high-dimensional state vector.

To avoid these disadvantages, we propose a hybrid particle-ensemble Kalman filter (based on the hybrid grid-particle filter of Salman [2]). Generally, the state vector will include both the flow variable and the drifter variable, and the distributions will be approximated by a weighted sum of particles. The distribution on the drifter variables will be updated according to the particle filter analysis step, but the distribution on the flow variables will be updated according to the ensemble Kalman filter. This method is more computationally intensive than a basic EnKF, but will approximate the highly non-Gaussian distributions on the drifters better than an EnKF. We present results of this hybrid method applied to the shallow water equations, a common model used in testing Lagrangian data assimilation schemes.

References

[1] C. Snyder et al. "Obstacles to high-dimensional particle filtering," ***Monthly Weather Review*****, vol. 136, pp. 4629-4640, December 2008. (Journal Article)**

[2] H. Salman. "A hybrid grid/particle filter for Lagrangian data assimilation. I: Formulating the passive scalar approximation," ***Q. J. R. Meteorol. Soc.*****, vol. 134, pp. 1539-1550, 2008. (Journal Article)**