Error Dynamics and Instability in Data Assimilation

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In many applications of data assimilation the observation operator, which maps from state space to observation space, leads to an ill-posed equation, meaning that the assimilation problem is unstable to small perturbations in the data. The use of a prior or background estimate acts to regularize the problem and thus ensure its stability at each assimilation time. This can be shown to be equivalent to a form of Tikhonov regularization as used in traditional inverse problems. Several common data assimilation algorithms, including variational methods and Kalman filters, can be considered in this framework.

In operational weather forecasting the data assimilation problem is usually cycled, so that the background state for one assimilation time is provided by a forecast of the analysis from the previous assimilation time. In this work we examine the stability of the error in a sequence of analyses as the assimilation process is cycled in time. Since the data assimilation problem is often solved in very high dimensional systems (of order 10^8 and higher), we derive theory using an infinite-dimensional framework.

We show that for a certain class of linear model dynamics it is possible to guarantee the stability of the analysis error in time by applying a multiplicative inflation to the background error variances. In the case of time-varying dynamics the inflation factor can be chosen adaptively at each assimilation time to ensure stability. However, as the inflation factor is increased, the assimilation problem at each time is less well-conditioned and the bound on the analysis error increases [1]. For nonlinear dynamics similar stability results are obtained under certain Lipschitz continuity and dissapitivity assumptions on the dynamical operator [1], [2]. The theory is illustrated with numerical results.

References

[1] Moodey, A.J.F. "Instability and regularization for data assimilation," *PhD thesis*, Department of Mathematics and Statistics, University of Reading, 2013. *(Submitted)*
[2] Moodey, A.J.F., Lawless, A.S., Potthast, R.W.E. and van Leeuwen, P.J. "Nonlinear error dynamics for cycled data assimilation methods," *Inverse Problems*, vol 29, pp. 025002, 2013