Pontryagin's Minimum Principle Applied to Meteorological Data Assimilation

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Pontryagin's Minimum Principle (PMP) came into prominence during the1950s – 1960s when space travel became a dominant theme [1]. A typical problem centered on minimizing the travel time or energy used by a space vehicle between two specified points by adding feedback control to the dynamical constraints — essentially taking account of the difference between the vehicle's position and a desired position determined *a priori*. Instead of the classical Lagrangian approach which describes the optimal trajectory as the solution of a second-order PDE, PMP using the Hamiltonian approach describes the optimal trajectory as the solution of first-order equations. Computationally, the optimal control and the optimal trajectory are obtained as a solution of a nonlinear two point boundary value problem (TPBVP) which can be solved iteratively by one of many known methods such as the shooting method, gradient methods, to name a few.

When the model and observations are linear, it can be shown using the sweep method, that the TPBVP reduces to two initial value (IVP) problems - one of which takes the form of a nonlinear matrix Riccati equation which is independent of the data and the other is a linear vector equation that depends on the data and uses the solution of the Riccati equation. Since the Riccati equation is independent of data, its solution can be computed off-line and can be used repeatedly.

In this research, we describe the first attempt at adapting the PMP to meteorological data assimilation [2] by optimally determining a forcing term (as a feedback control term) that minimizes the quadratic fit of model to noisy observations. Numerical tests of fit are performed on the spectral dynamics of the Burgers' equation — a linear form of the equation that captures the development of an extreme gradient and frontal-like structure. Results indicate that the feedback control terms produce excellent fit of model to observations. Further, the methodology is reconfigured to give information on model error [2].

References

[1] L. Pontryagin, V. Boltyanskii, R. Gamkrelidze, and E .Mischenko,(1962) *The Mathematical Theory of Optimal Control Processes*; Interscience Pub.,
[2] S. Lakshmivarahan, J. Lewis and D. Phan (2013) "Data Assimilation as Problem in Optimal Tracking: Application of Pontyagin's Minimum Principle to Atmospheric Sciences", **Journal of Atmospheric Sciences**, Vol 70, April Issue, pp1257-1277