

# The Ensemble Variational Assimilation and Bayesian Estimation

M. Jardak & O. Talagrand  
Laboratoire de Météorologie Dynamique  
École Normale Supérieure  
Paris, France

## Abstract

The purpose of assimilation can be stated as to determine the probability distribution for the state of the observed system, conditioned by the available data. In the case of linearity and additive Gaussian errors, a sample of independent realizations of the (Gaussian) conditional probability distribution can be obtained by repeating the following process. Perturb the data according to their own error probability distribution, and compute the Best Linear Unbiased Estimator (BLUE) corresponding to those perturbed data.

This approach has been implemented on two small dimension nonlinear chaotic systems, the Lorenz'96 and the Kuramoto-Sivashinsky equations, in the form of **Ensemble Variational Assimilation**, or **EnsVar**, in which the BLUE is determined by standard Variational Assimilation. The Bayesian character of a probability distribution cannot be in general objectively verified, and the weaker property of reliability (statistical consistency between predicted probabilities and observed frequencies of occurrence) is used instead.

The main conclusion is that EnsVar produces almost perfectly reliable, and accurate, ensembles. This conclusion remains valid for long assimilation periods, either through the use of Quasi-Static Variational Assimilation, in which the length of the assimilation window is progressively increased (in the case of a perfect model), or through weak-constraint assimilation (in the case of an imperfect model). In addition, non-Gaussianity of the errors has no significant impact.

Comparisons with Ensemble Kalman Filter and Particle Filters produce ensembles with a significantly lower reliability and lesser accuracy. On the other hand, the cost of EnsVar is higher.

The significance of those results is discussed in the general perspective of Bayesian estimation.